Solutions to JEE Main Home Practice Test - 9 | JEE - 2024

PHYSICS

SECTION-1

1.(C) $[\mathrm{A}]=[\mathrm{LT}^{-2}]$ or $[\mathrm{L}]=[\mathrm{AT}^2]$

$[\text{Work}]=[\text{Force}\times\text{Distance}]=[FL]=[FAT^2]$.

2.(B) $\lambda=\dfrac{h}{mv}$

$\lambda=\dfrac{h}{\sqrt{2meV}},\ \lambda'=\dfrac{h}{\sqrt{2MeV}}$

$\dfrac{\lambda'}{\lambda}=\sqrt{\dfrac{m}{M}}$ or $\lambda'=\lambda\sqrt{\dfrac{m}{M}}$

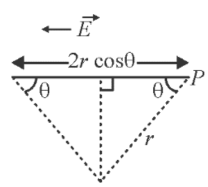
3.(B) The magnitude of electric field inside the cavity

$E_{cavity}=\dfrac{\rho a}{3\in_0}$. It is directed along a in radially outward direction

For touching the sphere again, electron must move a distance $2r\cos 45^\circ$ and time taken by electron for this is given as

$$t=\sqrt{\frac{2l}{\text{acceleration}}}$$

$$=\sqrt{\frac{2(\sqrt{2}r)}{eE/m}}$$



$(\because\ 2r\cos\theta=2r\times\dfrac{1}{\sqrt{2}}=\sqrt{2}r=\ell)$

$$\Rightarrow\quad t=\sqrt{\frac{6(\sqrt{2}\,mr\in_0)}{e\rho a}}$$

4.(B) $p=\sqrt{2mE_k}$

E_k is increased by a factor of 4. P becomes double. So, percentage increases in momentum is 100%

5.(D) $U=\dfrac{f}{2}RT;\ C_v=\dfrac{dU}{dT}=\dfrac{f}{2}R$

Now, $C_p=C_v+R=\dfrac{f}{2}R+R=\left(\dfrac{f}{2}+1\right)R$

$$\gamma=\frac{C_p}{C_v}=\frac{\left(\frac{f}{2}+1\right)R}{\frac{f}{2}R}=1+\frac{2}{f}$$

6.(A) $V=\omega\sqrt{a^2-\xi^2}=a\omega\dfrac{\sqrt{a^2-\xi^2}}{a}=v\dfrac{\sqrt{a^2-\xi^2}}{a}$

When $\xi=\dfrac{a}{2}$, then $V=v\times\dfrac{\sqrt{3}}{2}=\dfrac{1.732}{2}v=0.866v$

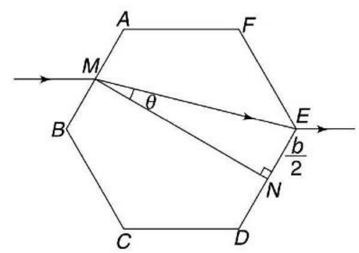
7.(D) P = F. v = constant

When F = μmg, net force on block becomes zero.

i.e. it has maximum or terminal velocity

$$\therefore \quad P = (\mu mg) v_{\max} \text{ or } v_{\max} = \frac{P}{\mu mg}$$

8.(A) For light beam to emerge parallel to the incident direction it must strike the face DE which is parallel to face AB. It means incident ray at M must go to E (or below it). Let side length of hexagon be b.



$MN = \sqrt{3}b$

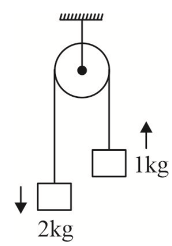
$$\therefore \quad ME = \sqrt{\left(\frac{b}{2}\right)^2 + \left(\sqrt{3}b\right)^2} = \frac{\sqrt{13}}{2}b$$

Using Snell's law

$1.\sin 30° = \mu \sin\theta$

$$\Rightarrow \quad \frac{1}{2} = \mu \frac{1}{\sqrt{13}} \quad \Rightarrow \quad \mu = \frac{\sqrt{13}}{2}$$

9.(A) $T = \dfrac{2m_1 m_2}{m_1 + m_2} \cdot g \qquad = \dfrac{2\times 2\times 1}{3}10 = \dfrac{40}{3}N$



$P = \dfrac{F}{A}; \qquad \dfrac{40}{3\pi}\times 10^6 = \dfrac{T}{\pi r^2}$

$r^2 = \dfrac{40/3N}{40/3\times 10^6}; \qquad r = 10^{-3} = 1mm$

10.(A) $T \propto R^{3/2}$

$$\frac{\Delta T}{T} = \frac{3}{2}\frac{\Delta R}{R} \text{ or } \frac{\Delta T}{T}\times 100 = \frac{3}{2}\frac{\Delta R}{R}\times 100 = \frac{3}{2}\times\frac{0.01R}{R}\times 100 = \frac{3}{2}\% = 1.5\%$$

Clearly, it is an increases.

11.(A) Energy contained per unit volume through which electromagnetic wave is propagating is sum of energy of electric field and energy of magnetic field i.e.

$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

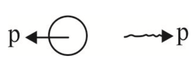
Where E and B are sinusoidal function

i.e., $E = E_0 \sin\omega(t - x/c)$ and $B = B_0 \sin\omega(t - x/c)$

$$u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 E_0^2 \sin^2\omega(t - x/c) = \frac{1}{2\epsilon_0}E_0^2\left[\frac{1-\cos 2\omega(t - x/c)}{2}\right]$$

The frequency of u_E is double than that of electromagnetic wave. Similarly, the frequency of u_B is double than that of electromagnetic wave.

12.(A)



$$\frac{E_{ph}}{c} = p, \; KE = \frac{p^2}{2m}$$

$$KE = \frac{E_{ph}^2}{2mC^2} = \frac{(3.05)^2}{2\times 25\times 931.5} = \frac{(3.05)^2}{50\times 931.5} = 0.2\,keV$$

13.(A) $\dfrac{\text{Rotational } K.E.}{\text{Translation } K.E.} = \dfrac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2} = \dfrac{mK^2\frac{v^2}{R^2}}{mv^2} = \dfrac{K^2}{R^2}$

14.(D) $I = \dfrac{Power}{hc/\lambda} \times \eta \times e \qquad = \dfrac{10^{-3}}{1240/456eV} \times 5 \times 10^{-3} \times e$

$I \approx 2 \times 10^{-6} A = 2\mu A$

15.(C) Final surface energy = 1000 × $4\pi r^2\sigma$

Initial surface energy E = $4\pi R^2\sigma$

Again, $\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$ or $R = 10r$

Now, $\dfrac{1000 \times 4\pi r^2\sigma}{4\pi R^2\sigma} = \dfrac{10 \times 4\pi R^2\sigma}{E} = 10E$

16.(D) $V - 12 = IR$

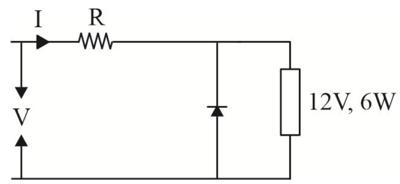
At initiation of break down $\Rightarrow$ V = 102V

Current in diode = 0A

$I = \dfrac{P}{V'} = \dfrac{6}{12} = 0.5A$

$102 - 12 = 0.5R$

$R = 180\Omega$



17.(A) $4 \propto (a - l)$ …(1)

$5 \propto (b - l)$ …(2)

$9 \propto (L - l)$ …(3)

Dividing (1) by (2), we get $\dfrac{4}{5} = \dfrac{a-l}{b-l}$

or $4b - 4l = 5a - 5l \qquad l = 5a - 4b$

Again, dividing (1) by (3), $\dfrac{4}{9} = \dfrac{a-l}{L-l}$ or $4L - 4l = 9a - 9l$

or $\qquad 4L = 9a - 5l$ *or* $4L = 9a - 5(5a - 4b)$ $\qquad$ or $\qquad 4L = 9a - 25a + 20b$

or $\qquad 4L = 20b - 16a$ or $L = 5b - 4a$

18.(B) $\frac{1}{2}PQ = |\vec{P} \times \vec{Q}| = PQ\sin\theta$ or $\sin\theta = \frac{1}{2}$ or $\theta = 30°$

19.(D) $v = \dfrac{1}{2 \times 3.14\sqrt{8 \times 0.5 \times 10^{-6}}} Hz \qquad \left(\because v = \dfrac{1}{2\pi\sqrt{LC}}\right)$

$= \dfrac{10^3}{4 \times 3.14} Hz = 79.6 Hz$

So, (B) is ruled out

Again, $\omega = \dfrac{1}{\sqrt{8 \times 0.5 \times 10^{-6}}} rad\ s^{-1} = \dfrac{10^3}{2} rad\ s^{-1} = 500\ rad\ s^{-1}$

20.(B) Let g_1 be the retardation due to gravity in the presence of air resistance. Clearly, $|g_1| > |g|$.

If v be the velocity of throw, then

Time of ascent, $t_1 = \dfrac{v}{g_1}$ … (i)

Maximum height $= \dfrac{v^2}{2g_1}$

Let g_2 be the acceleration due to gravity in the presence of air resistance. Clearly $|g_2| < |g|$

Now, $\dfrac{v^2}{2g_1} = \dfrac{1}{2} g_2 t_2^2$, where t_2 is the time of descent.

Now, $t_2^2 = \dfrac{2v^2}{2g_1 g_2} = \dfrac{v^2}{g_1 g_2}$ *or* $t_2 = \dfrac{v}{\sqrt{g_1 g_2}}$ …… (ii)

Equation (ii) divided by equation (i) gives $\dfrac{t_2}{t_1} = \dfrac{v}{\sqrt{g_1 g_2}} \times \dfrac{g_1}{v} = \dfrac{g_1}{\sqrt{g_1 g_2}} = \sqrt{\dfrac{g_1}{g_2}}$

But $|g_1| > |g|$ and $|g_2| > |g|$ $\quad\therefore\quad \sqrt{\dfrac{g_1}{g_2}} > 1 \quad\therefore\quad \dfrac{t_2}{t_1} > 1$ or $t_2 > t_1$ or $t_1 < t_2$

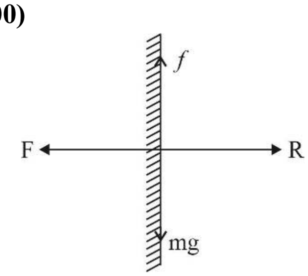
SECTION - 2

1.(60) If ΔW is the work done by the gas, then $\Delta U = \Delta Q - \Delta W$. That leads to 100 – 60 = – 20 + x (Since U_{AC} is unique). That leads to x = 60 cal.

2.(200) $\tau = I\alpha,\ \tau = \dfrac{I\omega_0}{t},\ t = \dfrac{I\omega_0}{\tau}$ or $\omega_0 = \dfrac{\tau t}{I} = \dfrac{10\times 10}{0.5} = 200 \text{ rad } s^{-1}$

3.(10) Dynamic resistance $= \dfrac{0.70-0.65}{5\times 10^{-3}}\Omega = \dfrac{0.05}{5\times 10^{-3}}\Omega = \dfrac{5\times 10^{-2}}{5\times 10^{-3}}\Omega = 10\Omega$

4.(1000)



$f = mg$; $\mu R = mg$; $\mu F = mg$

$F = \dfrac{mg}{\mu} = \dfrac{50\times 10}{0.5} N = 1000$

5.(240) $a = \dfrac{g\sin\theta}{1+\dfrac{2}{3}} = \dfrac{3}{5} g\sin\theta = \dfrac{3}{5}\times 10 \times \dfrac{2}{5} = \dfrac{12}{5} m\,s^{-2} = 2.4 ms^{-2}$

6.(8) In the first case, $\dfrac{1}{2} m v_{\max}^2 = 2h\nu_0 - h\nu_0 = h\nu_0$

In the second case, $\dfrac{1}{2} m v_{\max}'^2 = 5h\nu_0 - h\nu_0 = 4h\nu_0$

Clearly, $v_{\max}$ is doubled.

7.(100) Clearly, $E_2 = IY$

$$2 = \frac{12}{500+Y}Y \; or \; 500 + Y = 6Y \;\text{ or }\; 5Y = 500 \;\text{ or }\; Y = 100\Omega$$

8.(2) Field inside the cylinder, $E_{in}(r) = \dfrac{\rho r}{2\varepsilon_0}$

Therefore, $E\left(\dfrac{R}{2}\right) = \dfrac{\rho R}{4\varepsilon_0}$

Field outside the cylinder, $E_{out}(r) = \dfrac{\rho R^2}{2\varepsilon_0 r}$

Therefore, $E(2R) = \dfrac{\rho R}{4\varepsilon_0}$

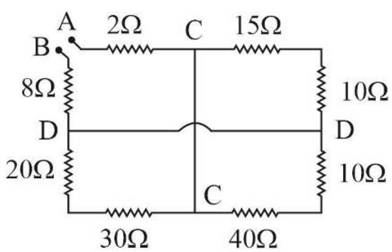
$$\frac{E(R/2)}{E(2R)} = 1 = \frac{2}{x} \quad \therefore \quad x = 2$$

9.(167) In order to have a good load regulations, the value of series resistance R should be such that the current through the Zener diode is much larger than the load current. For this, we select the Zener current five times the load current i.e., $I_Z = 5 \times 4 = 20mA.$ The total current through series resistance $R = I_Z + I_L = 20 + 4 = 24mA.$

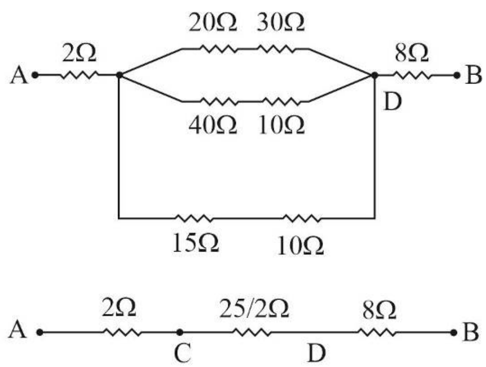
The voltage drop across $\mathrm{R} = 10.0 - 6.0 = 4.0V$

Therefore, $R = \dfrac{4.0V}{24mA} = 167\Omega$

10.(20)



Redrawing the circuit

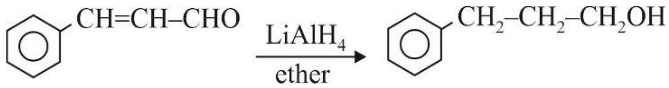


$R_{AB} = 22.5\Omega$

CHEMISTRY

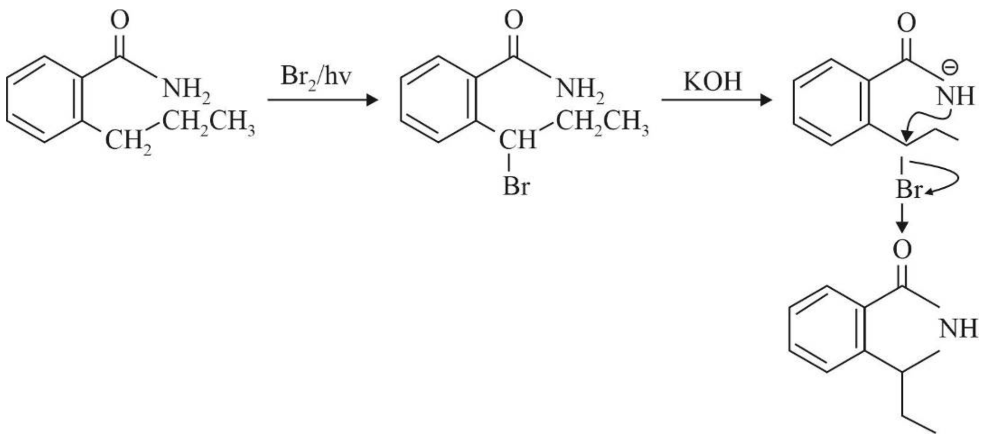
SECTION - 1

1.(C) Metamers are compounds which have different alkyl groups present along both side of polyvalent functional group.

2.(B) 

$LiAlH_4$ is R.A. it reduces aldehyde into alcohol. $LiAlH_4$ reduces double bond if conjugated with phenyl group in α,β unsaturated carbonyl compound along with carbonyl group. But in second case, it will not reduce carbon-carbon double bond.

$$CH_3-CH=CH-CHO \xrightarrow[\text{ether}]{LiAlH_4} CH_3CH=CHCH_2OH$$

3.(C) 

4.(A) Reactions,

(i) $Fe(s) \longrightarrow Fe^{2+} + 2e^-$; $E^o = +0.44\,V$ and $\Delta G_1^o = -nE^oF = -2\times 0.44\times F$

(ii) $2H^+ + 2e^- + \frac{1}{2}O_2 \longrightarrow H_2O(\ell)$; $E^o = +1.23\,V$ and $\Delta G_2^o = -2\times(+1.23)\times F$

Net reaction,

$$Fe(s) + 2H^+ + \frac{1}{2}O_2 \longrightarrow Fe^{2+} + H_2O(\ell)\ ;\ \Delta G_3^o = \Delta G_1^o + \Delta G_2^o$$

$$= [-2\times(+0.44) + (-2\times 1.23)] \quad = [-0.88 - 2.46]$$

$$= -3.34\,F = -3.34\times 96500\,J \quad = -322.32\,kJ/mol = -322\,kJ\ mol^{-1}$$

5.(A) $\log k(\min^{-1}) = 5 - \frac{2000}{T}$

$$\log_{10} k = \log_{10} A - \frac{E_a}{2.303RT}$$

I. $\log A = 5,\ A = 10^5$ is true

II. $\frac{E_a}{2.303R} = 2000$

$E_a = 2000\times 2.303\times 0.002\,kcal = 9.212\,kcal$, true

III. Equation represents straight line, hence true

6.(D) Ag^+ ion concentration required for precipitation for AgBr

$$\left[Ag^+\right]=\frac{K_{sp}\left(AgBr\right)}{\left[Br^-\right]}=\frac{5\times10^{-13}}{0.01}=5\times10^{-11}\,M$$

For Ag_2SO_4

$$\left[Ag^+\right]^2=\frac{K_{sp}\left[Ag_2SO_4\right]}{\left[SO_4^{2-}\right]}=\frac{2\times10^{-6}}{0.01}=2\times10^{-4}\,M^2$$

$$\left[Ag^+\right]=\left(2\times10^{-4}M^2\right)^{1/2}=1.4\times10^{-2}\,M$$

AgBr will precipitate first because it required low conc. of Ag^+.

7.(C) $\left[Xe\right]4f^{14}5d^{10}6s^2 6p^3$

The element E is Sn and the diagonal element of 6^{th} period is Bi. Its electronic configuration will be $\left[Xe\right]4f^{14}5d^{10}6s^2 6p^3$

8.(A) Ether do not react with metallic sodium.

9.(C) $\left[Ni\left(NH_3\right)_6\right]^{2+}$ has two unpaired electrons hence $\mu=\sqrt{8}=2.83BM$

$\left[Co\left(ox\right)_3\right]^{3-}$ has no unpaired electrons hence $\mu=0$

10.(B) $\dot{\ddot{N}}\ddot{O}\!:$ $\Rightarrow$ Odd e^- $\Rightarrow$ paramagnetic

$:\overleftarrow{C\equiv O}:$ $\Rightarrow$ Diamagnetic (isoelectronic with N_2)

MnO_4^{2-}, i.e. Mn^{6+} $\Rightarrow$ one unpaired electron $\Rightarrow$ paramagnetic

$$B_2,\text{i.e. } \sigma\left(1s\right)^2\sigma^*\left(1s\right)^2\sigma\left(2s\right)^2\sigma^*\left(2s\right)^2\begin{cases}\pi\left(2P_x\right)^1\\ \pi\left(2p_y\right)^1\end{cases}\Rightarrow \text{paramagnetic}$$

11.(C) According to Dalton's law,

$$p_T=p_B^o\chi_A+p_B^o\chi_B$$

$$550=p_B^o\times\frac{1}{4}+p_B^o\times\frac{3}{4}$$

Thus, $p_A^o+3p_B^o=2200$ ...(i)

When, 1 mole of B is further added to the solution

$$560=p_A^o\times\frac{1}{5}+p_B^o\times\frac{4}{5}$$

Thus, $p_A^o+4p_B^o=2800$ ...(ii)

On subtracting equation (ii) and (i)

$$p_B^o=2800-2200=600\,\text{mm Hg}$$

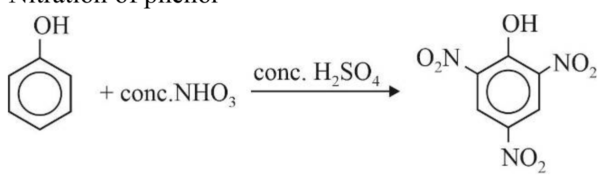
Putting the value of p_B^o in equation (i)

$$p_A^o=2200-3\times600;\;\; p_A^o=400\text{ mm Hg}$$

12.(C) $N_2O_5 \longrightarrow \underset{sp}{[NO_2]^+} \; \underset{sp^2}{[NO_3]^-}$

13.(B) Copper exhibits only +2 oxidation state in its stable compound because +2 state compounds of copper are formed by exothermic reaction.

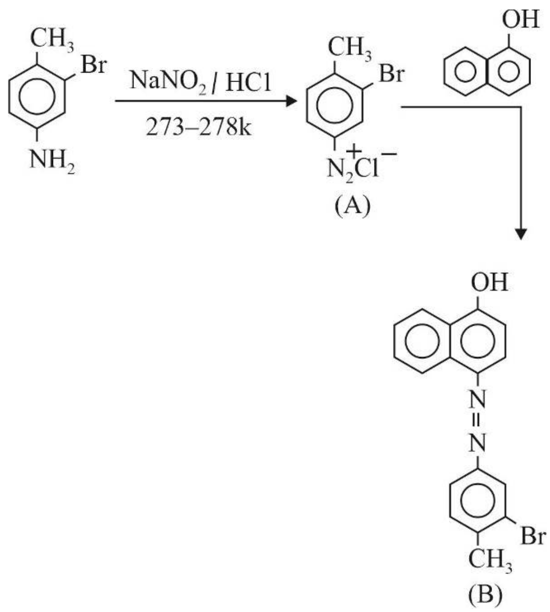
14.(D) Nitration of phenol



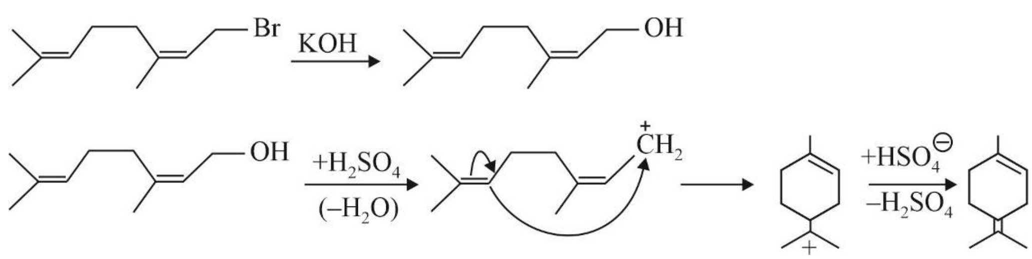
Picric acid

$$H_2SO_4 + HO-NO_2 \longrightarrow HSO_4^- + H_2\overset{+}{O}-NO_2 \longrightarrow H_2O + {}^{\oplus}NO_2$$

15.(A)



16.(A)



17.(C) $Sn + \text{conc. } HNO_3 \rightarrow H_2SnO_3 + 4NO_2 + H_2O$

18.(B) $C_xH_yN_z + CuO \rightarrow xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + Cu$

19.(B) The structure given in option (B) is of sucrose which is a non-reducing sugar.

20.(A) The leaving tendency of group is in the order of $NH_2^- < RO^- < RCOO^- < Cl^-$. The substrate having good leaving grp is more reactive towards nucleophilic substitution reaction

SECTION - 2

1.(1129) $\Delta_r G^\circ = \Delta_r H^\circ - T \times \Delta_r S^\circ$

$\Delta_r S^\circ = 2\times 81 - 4\times 24 - 3\times 205\ \text{J/mol}$

$\therefore \quad \Delta_r H^\circ = -2258.1\ \text{kJ/mol}$; $\Delta_r H^\circ = 2\times \Delta_f H^\circ(Cr_2O_3, S)$

$\therefore \quad \Delta_f H^\circ(Cr_2O_3, s) = -\dfrac{2258.1}{2} = -1129.05\text{kJ/mol}$

2.(2) $\Delta E = \dfrac{hc}{\lambda} = 6.52\times 10^{-18}\ \text{J}$

$\Delta E_H = \dfrac{3}{4}\times 2.176\times 10^{-18}\times z^2 = 1.63\times 10^{-18} z^2 \text{J}$

$Z^2 = \dfrac{\Delta E}{\Delta E_H} = 4 \quad \Rightarrow \quad Z = 2$

3.(279) Mass of $CHCl_3 = d\times V = 11.95\times 100 = 1195\text{g}$

Moles of $CHCl_3 = \dfrac{1195}{119.5} = 10$ mole

Moles of non-volatile solute = $\dfrac{2}{40} = \dfrac{1}{20}$

$\dfrac{P^\circ - P}{P^\circ} = \dfrac{n_B}{n_A} = \dfrac{\frac{1}{20}}{10}$; $\quad \dfrac{280 - P}{280} = \dfrac{1}{200}$

$280 - P = \dfrac{280}{200} = 1.4$; $\quad P = 280 - 1.4 = 278.6\text{mm Hg}$

4.(685) Moles of HCl = $\dfrac{M\times V}{1000} = \dfrac{5\times 3.26\times 1000}{1000} = 16.3$

Moles of $MgCO_3 = \dfrac{16.3}{2} = 8.15$

Mass of $MgCO_3 = 8.15\times 84 = 684.6\text{g}$

5.(3) $P_1 = 756\text{mm Hg}$

$P_2 = 760\text{mm Hg}$

$V_1 = 48.6\text{ml} \qquad V_2 = ?$

$T_1 = 300\text{K} \qquad T_2 = 273\text{k}$

Applying general gas equation $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$

$V_2 = \dfrac{P_1V_1T_2}{T_1P_2} = \dfrac{756\times 48.6}{300}\times\dfrac{273}{760} = 44\text{ml}$

Mass of organic compound = 0.305gm

% of $N_2 = \dfrac{28}{22400}\times V_2\times\dfrac{1}{0.305}\times 100 = \dfrac{28}{22400}\times 44\times\dfrac{1}{0.305}\times 100 = 18.03 \approx 18$

Hence, $\dfrac{\text{Nitrogen\%}}{6} = 3$

6.(11) $$Na_2CO_3 + HCl \longrightarrow NaCl + NaHCO_3$$

	Na_2CO_3	HCl	$NaCl$	$NaHCO_3$
Meq. before reaction	$\frac{95.4}{106}\times 1000$ = 900	150×1 = 150	0	0
Meq after reaction	750	0	150	150

The solution contains Na_2CO_3 and HCO_3^- and thus, acts as buffer

$$\therefore\ pH = -\log K_a + \log\frac{[CO_3^{2-}]}{[HCO_3^-]}$$

$$pH = -\log K_a + \log(750/150)$$

$$pH = -\log(2\times 10^{-11}) + \log 5 \quad = 10.7 + 0.7 = 11.4$$

7.(8) $$A_2(g) \longrightarrow B(g) + \frac{1}{2}C(g)$$

	$A_2(g)$	$B(g)$	$C(g)$
t = 0,	100	–	–
t = 5	100–x	x	x/2

Since, 100 – x + x + x/2 = 120

$\Rightarrow \quad x = 40 \quad \therefore \quad$ Rate of disappearance of $A_2(g) = \frac{-d[A_2]}{dt} = \frac{40}{5} = 8\text{mm min}^{-1}$

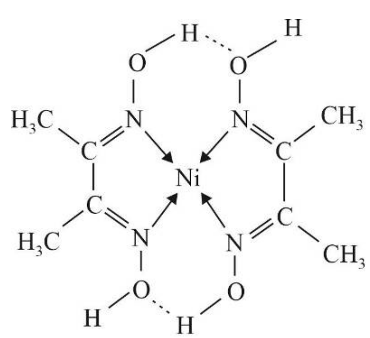
8.(1) $$M_2CO_3(s) \rightleftharpoons M_2O(s) + CO_2(g)$$

$$\therefore\ K_p = P_{CO_2} = 5\,atm$$

$$P_{CO_2} + P_{gas} = 6$$

$$P_{gas} = 6 - 5 \Rightarrow 1$$

9.(2)



Ni-DMG complex

Oxidation state of Ni is +2 and hybridization is dsp^2. $\mu = 0$ (no unpaired electron) hence, diamagnetic.

10.(50) $$2H_2O + 2e^{(-)} \xrightarrow{\text{electrolysis}} H_2 + 2OH^-$$

$$\text{moles of } OH^- \text{ produced} = \frac{50\times 8685}{96500} = 4.5$$

moles of acid present = 2.25

Mass present = 261gm; Percentage = 50%

MATHEMATICS

SECTION-1

1.(B) Coefficient of x and x^2 are ${}^{10}C_1 = {}^{10}C_2 - {}^{n}C_1$

$10 = 45 - n \quad \Rightarrow \quad n = 35$

2.(B) $\tan\left(2\tan^{-1}\left[\dfrac{\frac{3}{5}+\frac{3}{4}}{1-\frac{9}{20}}\right]\right)$; $\tan\left(2\tan^{-1}\left(\dfrac{27}{11}\right)\right)$; $\left(\dfrac{\frac{54}{11}}{1-\frac{729}{121}}\right) = \dfrac{-297}{304}$

3.(C) By similar ΔS OAP and ΔPDA

$AD = \dfrac{AO.\,PA}{OP} = \dfrac{\sqrt{8}}{3}; \qquad PD = \dfrac{8}{3}$

Area $\Delta ABP = \dfrac{8}{3} \times \dfrac{\sqrt{8}}{9} = \dfrac{16\sqrt{2}}{9}$

2nd Method

Area $= \dfrac{1}{2} PA.\,PB \sin 2\theta$; $\qquad \sin\theta = \dfrac{1}{3} \Rightarrow \cos\theta = \dfrac{2\sqrt{2}}{3}$

$= \dfrac{1}{2} 8.2 \sin\theta\cos\theta = 8.\dfrac{1}{3} \times \dfrac{2\sqrt{2}}{3} = \dfrac{16\sqrt{2}}{9}$

4.(B) Given that $y = \dfrac{x^2 + \alpha}{x+1} \Rightarrow x^2 - yx + \alpha - y = 0$

$\Rightarrow y^2 - 4(\alpha - y) \geq 0 \Rightarrow y^2 + 4y + 4 \geq 4\alpha + 4$

$4\alpha + 4 \leq 0 \Rightarrow \alpha \leq -1$

5.(C) If a is any number for which sin $a \neq 0$, then we don't get any fix tendency of $f(x)$ to approaches where $x \to a$

So, $\lim_{x \to a} f(x)$ does not exist

But if $\sin a = 0, \lim_{x \to a} f(x) = 0$

6.(B) $\displaystyle\int_{-\pi/2}^{\pi/2} 1\, dx \quad \left(\text{as} \lim_{t \to 0}\left[\frac{\sin t}{t}\right] = 0\right) = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

7.(A) $z = \left(e^{i\frac{\pi}{2}}\right)^{3i} = e^{-\frac{3\pi}{2}} \quad \Rightarrow \quad \text{Arg z} = 0$

8.(C) Equation is like $f(x) = f^{-1}(x)$

$\therefore$ Hence solution should be $f(x) = x$

$f(x) = x^2 + 5x + 4 = x$

$x^2 + 4x + 4 = 0 \quad \Rightarrow \quad \boxed{x = -2}$

9.(A) Here $x_1x_2x_3 = 2^2 \times 3 \times 5$

Let number of two's given to each of x_1, x_2, x_3 be a, b, c

Then $a+b+c=2,\ a, b, c \geq 0$

The number of integral solutions of this equation is equal to coefficient of x^2]in $(1-x)^{-3}$ i.e., ${}^4C_2$

i.e. the available 2 two's can be distributed among x_1, x_2 and x_3 in ${}^4C_2 = 6$ ways

Similarly, the available 1 three can be distributed among x_1, x_2, x_3 in ${}^3C_2 = 3$ ways

(= coefficient of x in $(1-x)^{-3}$)

$\therefore$ Total number of ways $= {}^4C_2 \times {}^3C_2 \times {}^3C_2 = 6\times3\times3 = 54$ ways

10.(D) $P\left(\overline{A}\cap B\right)+P\left(A\cap\overline{B}\right)=\alpha$; $\quad P\left(\overline{B}\cap C\right)+P\left(B\cap\overline{C}\right)=2\alpha-1$

$P\left(\overline{A}\cap C\right)+P\left(A\cap\overline{C}\right)=\alpha$; $\quad P\left(A\cap B\cap C\right)=(1-\alpha)^2$

$P(A)+P(B)-2P(A\cap B)=\alpha$ ... (i)

$P(B)+P(C)-2P(B\cap C)=\alpha$ ... (ii)

$P(C)+P(A)-2P(A\cap C)=2\alpha-1$ ... (iii)

Equation (i), (ii), (iii)

$$P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)=\frac{4\alpha-1}{2}$$

So $P(A\cup B\cup C)=\dfrac{4\alpha-1}{2}+(\alpha-1)^2=\dfrac{4\alpha-1+2(\alpha-1)^2}{2}$

$$=\frac{4\alpha-1+2\alpha^2+2-4\alpha}{2}=\frac{2\alpha^2+1}{2}=\alpha^2+\frac{1}{2}$$

$\alpha^2+\dfrac{1}{2}=\dfrac{3}{4}$ $\quad$ (Given $P(A\cup B\cup C)=\dfrac{3}{4}$); $\quad \alpha=\dfrac{1}{2}$

11.(B) $f(x)=2x^3-3x^2-f''(3)x-6x+f''\left(\dfrac{1}{2}\right)=0$

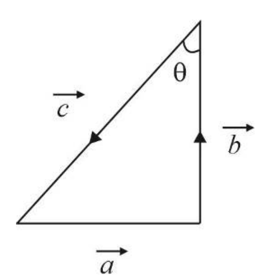
$f'(x)=6x^2-6x-f''(3)-6$; $\quad f''(x)=12x-6$

$f''(3)=36-6=30$; $\quad f''\left(\dfrac{1}{2}\right)=0$

So $f'(x)=6\left(x^2-x-6\right)=6(x-3)(x+2)$

Clearly $f(x)$ is decreasing in $(-2,3)$

12.(C) $\overline{a}+\overline{b}=-\overline{c}$; $\quad a^2+b^2+2\overline{a}.\overline{b}=c^2$



$a^2+b^2=c^2$

i.e. $a^2=c^2-b^2$, also given $\dfrac{1}{c^2}+\dfrac{1}{b^2}=\dfrac{1}{a^2}$

$\dfrac{1}{c^2}+\dfrac{1}{b^2}=\dfrac{1}{c^2-b^2}$; $\quad \dfrac{b^2+c^2}{c^2b^2}=\dfrac{1}{c^2-b^2}$

$c^4-b^4=c^2b^2$; $\quad \dfrac{c^2}{b^2}-\dfrac{b^2}{c^2}=1$

Let $\frac{b^2}{c^2} = x$; $\frac{1}{x} - x = 1$; $x^2 + x - 1 = 0$

$x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{\sqrt{5}-1}{2}$ (x can not be –ve)

$\frac{b^2}{c^2} = \cos^2\theta = \frac{\sqrt{5}-1}{2}$ $\therefore$ $\left(\hat{b}.\hat{c}\right)^2 = \cos^2\theta = \frac{\sqrt{5}-1}{2}$

13.(B) Clearly $\therefore$ $|A| = e^x \sec y - 4x^2 \tan y$

$x^3 \frac{dy}{dx} - e^x \sec y + 4x^2 \tan y = 0$ on simplifying

$\cos y \frac{dy}{dx} + \frac{4}{x}\sin y = \frac{e^x}{x^3}$. Put $\sin y = t$ [linear $D.E$]

$\sin y = e^x(x-1)x^{-4} + C$; $y(1) = 0 \Rightarrow C = 0 \Rightarrow \sin y = e^x(x-1)x^{-4}$

14.(C) $2x + y + z - 5 = 0$ is the plane which contain ‘C’

15.(B) Mean $\frac{n(n+1)}{2n} = \frac{n+1}{2}$

Variance $\frac{\sum n^2}{n} - \left(\frac{n+1}{2}\right)^2 = \frac{n(n+1)(2n+1)}{6n} - \frac{(n+1)^2}{4} = \frac{(n+1)}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$

$\frac{(n+1)(4n+2-3n-3)}{12} \Rightarrow \frac{(n+1)(n-1)}{12} \Rightarrow \frac{n^2-1}{12} + \frac{n+1}{2} = 21$

$n^2 + 6n + 5 = 252 \Rightarrow n^2 + 6n - 247 = 0 \Rightarrow n = 13$

16.(D) $g\left(\frac{\pi}{2}\right) = \int_{-\pi/2}^{\pi/2} \sin\left(\frac{\pi}{2} + f(x)\right)dx$; $g\left(\frac{\pi}{2}\right) = \int_{-\pi/2}^{\pi/2} \cos\left(f(x)\right)dx$

$g\left(\frac{\pi}{2}\right) = 2\int_0^{\pi/2} \cos\left(f(x)\right)dx$ $\left(\cos\left(f(x)\right) \text{is even}\right)$

$\therefore$ $\int \cos\left(f(x)\right)dx = \frac{g\left(\frac{\pi}{2}\right)}{2}$

17.(C) $\frac{k-\lambda}{h-\lambda} = -1 \Rightarrow k - \lambda = -h + \lambda$ $\boxed{k + h = 2\lambda}$

and $2h = x + \lambda \Rightarrow x = 2h - \left(\frac{k+h}{2}\right)$

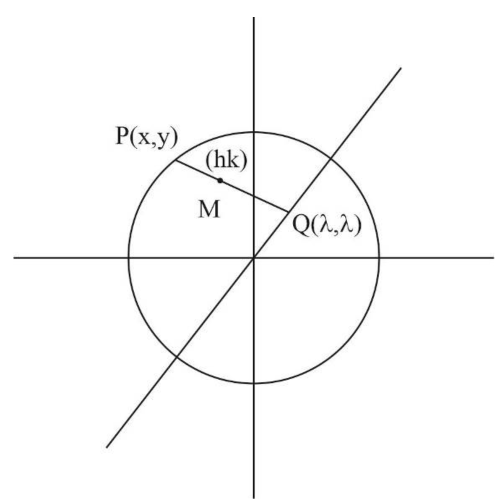
$2k = y + \lambda$ $y = 2k - \frac{k+h}{2}$

$\Rightarrow \frac{(3h-k)^2}{4} + \frac{(3k-h)^2}{4} = 1$

$9h^2 + k^2 - 6hk + 9k^2 + h^2 - 6hk = 4$

$10\left(h^2 + k^2\right) - 12hk = 4$

$5\left(h^2 + k^2\right) - 6hk = 2$; $5\left(x^2 + y^2\right) - 6xy = 2$



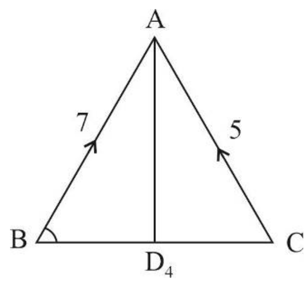
18.(D) $\lim_{h\to 0}\dfrac{f(2+5h^2-3h)-f(2)}{f(3+h^2-2h)-f(3)}$ by L. Hospital Rule

$$\lim_{h\to 0}\frac{f'(2+5h^2-3h)(10h-3)}{f'(3+h^2-2h)(2h-2)}=\frac{3}{4}$$

19.(C) $\log_a^x\left(2+3+4.....(n+1)\right)=n^2+3n$

$$\log_a^x\left(\frac{n}{2}(n+3)\right)=n(n+3) \qquad \Rightarrow \qquad \boxed{x=a^2}$$

20.(A) Projection of $\overrightarrow{BA}$ on $\overrightarrow{BC}$ is equal to $=\left|\overrightarrow{BA}\right|\cos\angle ABC \;=7\left|\dfrac{7^2+4^2-5^2}{2\times 7\times 4}\right|=5$



SECTION-2

1.(2) $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$; $\quad |A|=4$

$$\left|3adj\left(\lambda A^{-1}\right)\right|=\left|3\lambda^2 adj\left(A^{-1}\right)\right|=108 \quad\Rightarrow\quad \left(3\lambda^2\right)^3|A|^{-2}=108$$

$$\Rightarrow \qquad \frac{27\lambda^6}{|A|^2}=108 \quad\Rightarrow\quad \lambda^2=\frac{4\times 16}{27} \quad\Rightarrow\quad \lambda^6=64 \quad\Rightarrow\quad \lambda=2$$

2.(4) $25^{\left(2x-x^2+1\right)}+9^{\left(2x-x^2+1\right)}=34\cdot\dfrac{3^{2x-x^2+1}}{3}\cdot\dfrac{5^{2x-x^2+1}}{5}$

Let $3^{2x-x^2+1}=a$ and $5^{2x-x^2+1}=0$

$a^2+b^2=\dfrac{34}{15}ab$

$15a^2-34ab+15b^2=0\Rightarrow(3a-5b)(5a-3b)=0$

Case-1: If $\dfrac{a}{b}=\dfrac{5}{3}$

$$\Rightarrow \qquad \left(\frac{3}{5}\right)^{2x-x^2+1}=\frac{5}{3} \quad\Rightarrow\quad 2x-x^2+1=-1\Rightarrow x^2-2x-2=0$$

Case-2: If $\dfrac{a}{b}=\dfrac{3}{5}$; $\qquad \left(\dfrac{3}{5}\right)^{2x-x^2+1}=\dfrac{3}{5}$

$$\Rightarrow \qquad 2x-x^2+1=1\Rightarrow x=0 \text{ and } 2$$

Sum of all values of x is 4

3.(2) $\dfrac{dy}{dx}=xy(1+y)$; $\quad \displaystyle\int\frac{dy}{(1+y)y}=\int x\,dx$

$\dfrac{2y}{1+y}=e^{\frac{x^2}{2}} \quad (\because \; f(0)=1) \quad \Rightarrow \quad f(2)=\dfrac{e^2}{2-e^2}$

4.(2021) $(a^2+4a+3)x^2+(a^2-a-2)x+(a+1)a=0$

$\Rightarrow (a+1)(a+3)x^2+(a+1)(a-2)x+(a+1)a=0$

$(a+1)\left[(a+3)x^2+(a-2)x+a\right]=0$

If $a=-1$, then quadratic have more than two roots.

5.(7) Clearly line is altitude through A

$\dfrac{y-3}{x+2}=2$; $\quad y=2x+7$

6.(12) Slope of tangent at $(\alpha,\beta)=\dfrac{-\alpha}{\beta}$

Slope of normal through (3, 3/2) = $\dfrac{1}{2} \quad \Rightarrow \quad -\dfrac{\alpha}{\beta}=-2 \quad \Rightarrow \quad \alpha=2\beta$

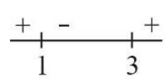
also $\quad \alpha^2+\beta^2=4$

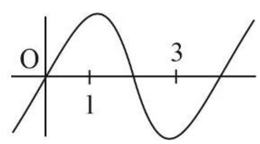
$\beta^2=\dfrac{4}{5}$; $\quad \alpha^2=\dfrac{6}{5}$; $\quad 5\left(\alpha^2-\beta^2\right)=12$

7.(8) $f(x)=x^3-6x^2+9x-3$

$f'(x)=3x^2-12x+9=3(x-1)(x-3)$

$f(1)=1\; f(3)=-3$





$g(x)=\begin{cases} f(x) & 0\le x\le 1 \\ 1 & 1\le x\le 3 \\ 4-x & 3<x\le 4 \end{cases}$ $\qquad$ Range of $g(x)[-3,1]$ $\;\therefore 3^2-1^2=8$

8.(19) Given $\dfrac{1}{\alpha(\alpha+1)....(\alpha+20)}=\displaystyle\sum_{k=0}^{20}\frac{A_k}{\alpha+k}$

$\dfrac{1}{(-2)(-1)1.2....18}=A_2 \qquad \dfrac{1}{(-4)(-3)(-2)(-1)(1)(2)....(16)}=A_4$

$\dfrac{1}{2!18!}=A_2 \qquad \dfrac{1}{4!16!}=A_4$

Similarly $\dfrac{1}{-3(-2)(-1)1.2.17}=A_3$; $\quad \dfrac{A_3+A_4}{A_2}=\dfrac{A_3}{A_2}+\dfrac{A_4}{A_2}=\dfrac{39}{2}$

$\dfrac{-1}{3!17!}=A_3 \qquad \therefore \left[\dfrac{A_3}{A_2}+\dfrac{A_4}{A_2}\right]=19$

9.(7) If a, x, y, z, b A.P.

$$x = \frac{3a+b}{4},\ y = \frac{a+b}{2} \text{ and } z = \frac{a+3b}{4}$$

If a, x, y, z, b H.P $x = \dfrac{4ab}{3b+a}, y = \dfrac{2ab}{a+b}$ and $z = \dfrac{4ab}{3a+b}$

If $\left(\dfrac{3a+b}{4}\right)\left(\dfrac{a+b}{2}\right)\left(\dfrac{a+3b}{4}\right) = 55$ and $\left(\dfrac{4ab}{3b+a}\right)\left(\dfrac{2ab}{a+b}\right)\left(\dfrac{4ab}{3a+b}\right) = \dfrac{343}{55} \Rightarrow ab = 7$

10.(2) $\displaystyle\lim_{x\to 0} \frac{ax\cos x + b\sin x}{x^2 \sin x} = \frac{1}{3}$

$$\lim_{x\to 0} \frac{ax\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}....\right) + b\left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right)}{x^2 \sin x} = \frac{1}{3}$$

$$a + b = 0 \ and\ -\frac{a}{2} - \frac{b}{6} = \frac{1}{3}$$

$\Rightarrow \quad b = 1, a = -1$